Abstract

In this paper a fuzzy Digital Bathymetric Model is presented, as well as the corresponding methods of analysis. The tolerance of the measuring instruments, the spatial and temporal densities of the measured data, the used methods of interpolation and the morphologic and morphodymanic characteristics of the area corresponding to the data set are the sources of uncertainty. These sources of uncertainty are encapsulated to construct fuzzy numbers that represent the data. The practicability is shown on bathymetric data sets that cover the offshore area of the island of Langeoog off the German coast of the north sea. Spatial-temporal fuzzy interpolation and analysis are shown to extend spatial-temporal interpolation to model the morphodynamic processes in a more comprehensive way.

Keywords: uncertainty, fuzzy number, spatial-temporal fuzzy interpolation, fuzzy Digital Bathymetric Model, morphodynamics.

1 Introduction

Basic data together with its corresponding procedures for spatial-temporal interpolation constitutes a new definition of a Digital Bathymetric Model. This definition provides a platform for new methods of analysis of large scale morphodynamic processes.

However, spatial and temporal inconsistency of measurements in addition to the uncertainty induced by the imprecision of measuring instruments result in a lack of knowledge and uncertainty about the basic data, which should be reflected on in the result of the analysis of the morphodynamic processes. Moreover, the method of interpolation introduces extra uncertainty in the resulting Digital Bathymetric Model. The adoption of a fuzzy approach, that takes this information into account, addresses directly this uncertainty and vagueness in the Digital Bathymetric Model and the subsequent analysis of morphodynamics, see Figure 1.

In [3] an object-oriented database-supported Digital Bathymetric Model with the associated interpretation rules was presented. Spatial-temporal interpolation methods and the optimization of the interpolation algorithms to handle non homogeneously distributed spatial and temporal measurements were discussed. Different procedures for a more adequate analysis of morphodynamics were tested.

This paper elaborates the uncertainty associated with the Digital Bathymetric Model proposed in [3] and discusses how this uncertainty can be incorporated in the model. The tolerance of the measuring instruments, the spatial and temporal densities of the measured data, the used methods for interpolation and the geometrical and morphodymanic characteristics of the area corresponding to the data set are the sources of uncertainty. These different sources of uncertainty and impreciseness are considered here to build a fuzzy Digital Bathymetric Model.

In the next section the sources of uncertainty in the process of building a spatial-temporal Digital Bathymetric Model are presented. The construction of fuzzy numbers from the sources of uncertainty is introduced in section 3. Section 4 is denoted to fuzzy interpolations. The different cases of fuzzy interpolation of measured data are discussed there. The fuzzy Digital Bathymetric Model in space and time is introduced in section 5. In section 6 the possible techniques of analysis of morphodynamics, that are presented in [3], and the one adopted here are highlighted. The proposed Digital Bathymetric Model has been implemented and the practicability is verified on bathymetric data sets in section 7. A conclusion and an outlook is considered in the last section.

The theory of fuzzy sets and fuzzy numbers is not subject of this paper, although a basic knowledge about this theory is a prerequisite for the thorough understanding of the text. For detailed treatment of the subject the references [1] and [2] are recommended. Throughout this paper any symbol with a tilde above, for instance, $\widetilde{x}$, refers to a fuzzy number or to a resulting fuzzy number in case of a fuzzy function, for example, $\widetilde{f}(x)$. Otherwise the symbols refer to crisp real numbers.

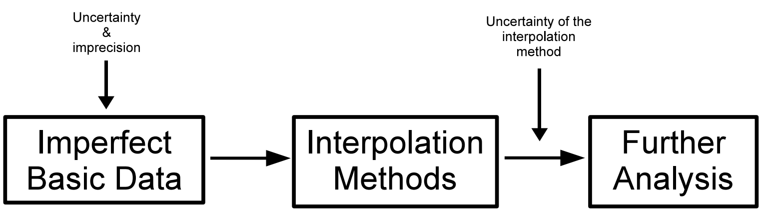

Figure 1: Sources of uncertainty and impreciseness.

2 Available Information and Sources of Uncertainty

Measurement data resulting from several surveying campaigns are the basic data sets that are used in building Digital Bathymetric Models. The fact that these measurements are taken at discrete, distributed points in time and space reduces the actual state of the measured continuous bathymetry. This introduces an uncertainty about the resulting bathymetric model.

The inadequate and imprecise representation of the sea floor, in addition to the genuine impreciseness in the basic data resulting from errors and inaccuracy of the acquisition method increases, if neglected, the uncertainty in the model.

Bathymetric models are based on data sets and the associated interpolation procedures. The assumption that the function resulting from the interpolation procedure represents the real morphology of the sea bed is flawed with great uncertainty.

The information available besides the actual basic data set may be listed as follows:

1. the tolerance of the data acquisition technique,
2. the fuzziness induced by the interpolation,
3. the spatial and temporal densities of the measured data, and
4. the morphologic and morphodynamic characteristics of the area corresponding to the data set

Based only on the basic data set, while neglecting the information mentioned above, one can build a precise but uncertain model. In order to decrease the uncertainty in the model one must incorporate these neglected pieces of information in the bathymetric model. Carrying all the information content available over to the analysis stage enables making more certain end conclusions about the morphologic changes of the sea floor.

3 From the Sources of Uncertainty to Fuzzy Numbers

The none traditional approach of fuzzy theory to handling the uncertainty of information is followed here to model the bathymetry of the sea floor. Fuzzy numbers are suitable for conglomerating all the available information in one entity and at the same time reflecting its uncertainty. The basic notion necessary here is the degree of presumption. The degree of presumption takes values between 0 and 1. A fuzzy number may be constructed depending on the degree of presumption or at least modified by altering the degree of presumption for every α-cut of the fuzzy number. Thereby, all the mentioned different kinds of uncertainty can be accounted for. Every aspect of the sources of uncertainty will be discussed here separately for more clarity and understanding.

3.1 The tolerance of the data acquisition technique

See floor surveying campaigns are usually conducted by different governmental and non-governmental research institutions. Therefore, many techniques are employed to collect information about the sea floor topography. These data acquisition techniques vary from single-beam, multi-beam and fan sonar to laser scanning. Every single technique has different sources of uncertainty and different ways to handle these uncertainties. Representing the resulting information by a fuzzy number accounts for the kind of uncertainty induced by applying the different techniques in a clear and consistent manner. It makes it possible to take the other uncertainties induced in the process of building the Digital Bathymetric Model into account in a more comprehensible way.

The displayed value of a measuring instrument, that can be assumed by a random variable, is generally not accurately ascertained. The probability distribution function of the random variable describes the measured value approximately or vaguely. A simple triangular distribution, that appoximates the non-linear unknown distribution function, is assumed. Therefore, the spam of the distribution function is given the degree of presumption of zero and the displayed value is given the degree of presumption of one. In between it is linearly interpolated to give the constructed fuzzy number. The uncertainty about the measured data is, thereby, described by a fuzzy number, see Figure 2. Uncertainty and impreciseness may well be in the information about the depth as well as the location.

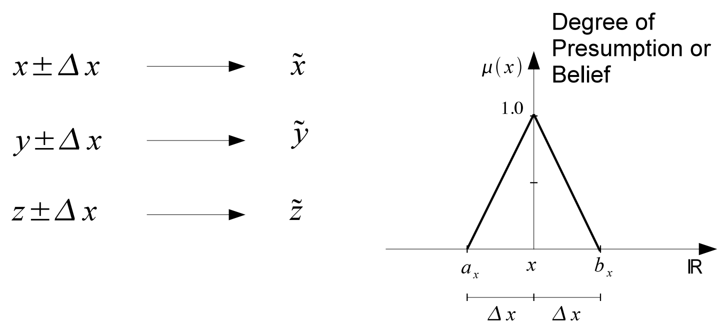

Figure 2: The fuzzy number representing the uncertainty in measured data

3.2 The Fuzziness induced by the interpolation

The set of data that makes a specific interpolating function is regarded as an incomplete information item. Interpolation methods are used to get a totally unknown piece of information from this incompletely known information item. In order to reduce the induced uncertainty about the interpolated value, a fuzzy quantification of it is conducted. The fuzzy quantification procedure is demonstrated on the basis of coordinate interpolation, see Figure 3. Let x_1 and x_2 be two pieces of information, each representing a specific location on the x axis. Considering the region between x_1 and x_2 as unknown, we could linearly interpolate the unknown location x ($x_1 \leq x \leq x_2$) easily.

Figure 3: Coordinate interpolation.

The interpolating function is given as

$$\overline{x} = \sum_{i=1}^{2} x \cdot \phi_i(x) \tag{1}$$

Nevertheless, having only x_1, x_2 and the interpolation method as an information item to describe the whole interval $[x_1, x_2]$ makes this information incomplete and induces an amount of uncertainty.

The fuzzy quantification of the interpolated value is given as a fuzzy number

$$\widetilde{x} = \{(\xi, \mu(\xi)) | \xi \in \mathbf{R}, \mu(\xi) \in [x_1, x_2] \to [0, 1]\} \tag{2}$$

The presumption function is given in quasi-LR-representation, see Figure 4. That means the function is divided into a left function and a right function and given as

$$\mu(\xi) = \begin{cases} L(\frac{x-\xi}{x-x_1}) & \text{for } \; x_1 \leq \xi < x \\ 1 & \text{for } \quad \xi = x \\ R(\frac{\xi-x}{x_2-x}) & \text{for } \; x < \xi \leq x_2 \end{cases} \tag{3}$$

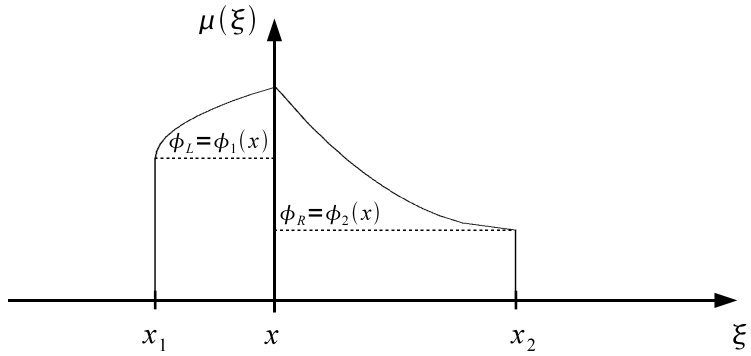

Figure 4: Quasi-LR-representation of a fuzzy number.

The left function is given as

$$L(\bar{\xi}) := (1 - \bar{\xi})^{\alpha_L}(1 - \phi_L) + \phi_L \;\; ; \;\; \alpha_L := \frac{1 - \phi_L}{\phi_L} \;\; ; \;\; \bar{\xi} = \frac{x - \xi}{x - x_1} \tag{4}$$

and the right function is given as

$$R(\bar{\xi}) := (1-\bar{\xi})^{\alpha_R}(1-\phi_R) + \phi_R \ ; \ \ \alpha_R := \frac{1-\phi_R}{\phi_R} \ ; \ \bar{\xi} = \frac{\xi - x}{x_2 - x} \tag{5}$$

The values of ϕ_L and ϕ_R are given by the basis functions ϕ_1 and ϕ_2, respectively.

Using the suggested quantification to interpolate the coordinate x between $x_1 = 0$ and $x_2 = 1$ gives the fuzzy numbers shown in Figure 5 at $x = 0.0$,$x = 0.25$,$x = 0.5$,$x = 0.75$ and $x = 1.0$.

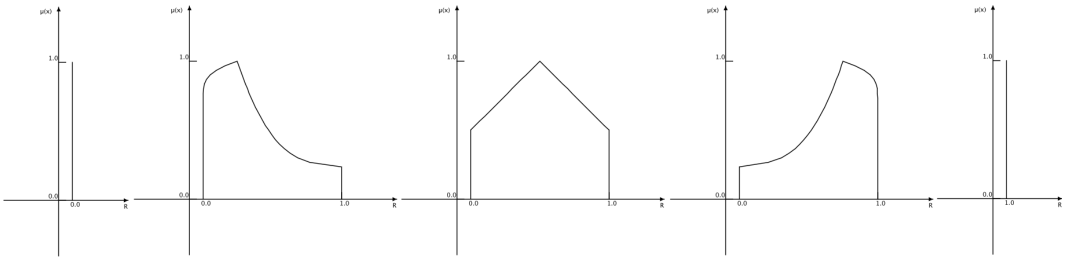

Figure 5: The interpolated fuzzy numbers at $x = 0.0$,$x = 0.25$,$x = 0.5$,$x = 0.75$ and $x = 1.0$ from left to right respectively.

Describing the sought information x as a fuzzy number $\widetilde{x}$, which is based on the degrees of presumption, increases the certainty about the interpolated value. This exploits all the used information content explicitly, which is otherwise represented only implicitly in the interpolating function. This procedure must be slightly modified in accordance with the applied interpolation method. However, the basic idea suggested here is generally valid.

3.3 The data density and the characteristics of the region

The degrees of presumption of the constructed fuzzy number can be modified by a Characteristic-Density Factor. This factor expresses the information content of the association between the characteristics of the area under study and the density of the measured data that cover this area. One has to differentiate between two cases. The first one is the morphologic characteristic associated with the spatial density and the second one is the morphodynamic characteristic associated with the temporal density. This association is essential, because having only the density of the data does not justify the attempt to modify the degrees of presumptions. Only in connection with the characteristic of the area is this attempt plausible.

The Characteristic-Density Factor influences the quantification of the fuzziness. This influence can be demonstrated in decreasing or increasing the precision without affecting the uncertainty about the modified information value. This effect can be done by using a fuzzy modification operator on the fuzzy number representing the fuzzy value.

$$mod[\mu(\xi)] = \mu^m(\xi) \tag{6}$$

where m is the Characteristic-Density factor.
The *morphological* Characteristic- *spatial* Density factor m_s could be given as

$$m_s = density^{|grad\, z(x,y)|}\ . \tag{7}$$

The *morphodynamic* Characteristic- *temporal* Density factor m_t could be given as

$$m_t = density^{|grad\, z(t)|}\ . \tag{8}$$

A suggested density function is given as

$$density = \sum_{i=1}^{n} D_i(p) \tag{9}$$

and

$$D_i(p) = e^{-distance(p,p_i)^2} \tag{10}$$

where p_i are the sampling points.

4 Fuzzy Interpolations

Considering a set of crisp data, such that at various points x_i there is a crisp information $f(x_i)$, the interpolation of such discrete crisp data in terms of relatively simple functions is well-grounded. The interpolation methods used are generally based on the simple form of an interpolation function

$$\overline{f}(x) = \sum_{i=1}^{n} f_i(x_i) \cdot \phi_i(x) \tag{11}$$

where the basis function $\phi_i(x)$ satisfies the interpolation condition:

$$\phi_i(x_k) = \begin{cases} 1 & \text{for} \quad k = i \\ 0 & \text{for} \quad k \neq i \end{cases} \tag{12}$$

A more general and systematic view of the interpolation methods that cover interpolating discrete fuzzy data is required. In the following four sorts of problems are distinguished. First of all the interpolation problem of data with fuzzy depth at a crisp location is presented. It is then followed by the problem of interpolating data with crisp depth and fuzzy location. The next interpolation problem is one in which the depth as well as the location are fuzzy. The last problem presented here deals with interpolating data with crisp depth at a crisp location taking into consideration the fuzziness induced by the interpolation method itself.

1. The interpolation problem of data with fuzzy depth at a crisp location

   In this case imprecise and uncertain depths $\widetilde{f}_i(x_i)$ are given at crisp locations x_i. The used basis functions are crisp functions here.

$$\widetilde{\widetilde{f}}(x) = \sum_{i=1}^{n} \widetilde{f}_i(x_i) \cdot \phi_i(x) \tag{13}$$

   Figure 6 shows a one-dimensional fuzzy linear interpolation of data with crisp locations x_1 and x_2 and fuzzy interpolated quantities $\widetilde{f}_1(x_1)$ and $\widetilde{f}_2(x_2)$, respectively.

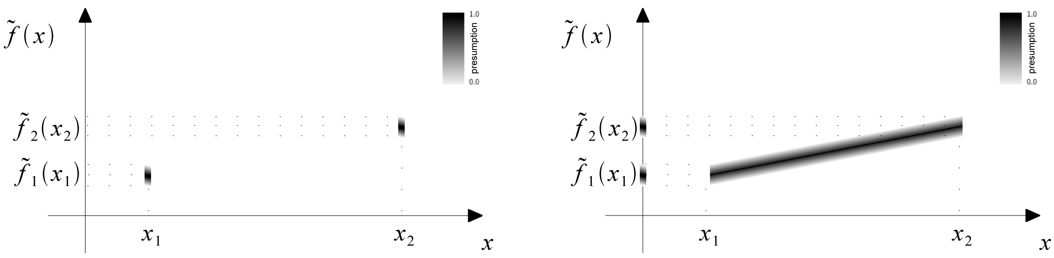

Figure 6: Fuzzy interpolation of data with crisp locations and fuzzy quantities.

2. The interpolation problem of data with crisp depth at a fuzzy location

   Here the fuzziness of the interpolant results from the impreciseness and uncertainty in the given locations $\widetilde{x}_i$, whereas the quantities $f_i(\widetilde{x}_i)$ at $\widetilde{x}_i$ are crisp. The resulting basis functions are fuzzy functions $\widetilde{\phi}_i(x)$

$$\widetilde{\widetilde{f}}(x) = \sum_{i=1}^{n} f_i(\widetilde{x}_i) \cdot \widetilde{\phi}_i(x) \tag{14}$$

   Figure 7 shows a one-dimensional fuzzy linear interpolation of data with fuzzy locations $\widetilde{x}_1$ and $\widetilde{x}_2$ and the respective crisp interpolated quantities $f_1(\widetilde{x}_1)$ and $f_2(\widetilde{x}_2)$.

3. The interpolation problem of data with fuzzy depth at a fuzzy location

   In the case where the impreciseness and uncertainty are in both the locations $\widetilde{x}_i$ and in the quantities $\widetilde{f}_i(\widetilde{x}_i)$ the used basis functions are fuzzy, too. The resulting fuzziness in the interpolating function is the overlapping of the fuzziness in both the location and the interpolated quantity.

$$\widetilde{\widetilde{f}}(x) = \sum_{i=1}^{n} \widetilde{f}_i(\widetilde{x}_i) \cdot \widetilde{\phi}_i(x) \tag{15}$$

   Figure 8 shows a one-dimensional fuzzy linear interpolation of data with fuzzy locations $\widetilde{x}_1$ and $\widetilde{x}_2$ and fuzzy interpolated quantities $\widetilde{f}_1(\widetilde{x}_1)$ and $\widetilde{f}_2(\widetilde{x}_2)$.

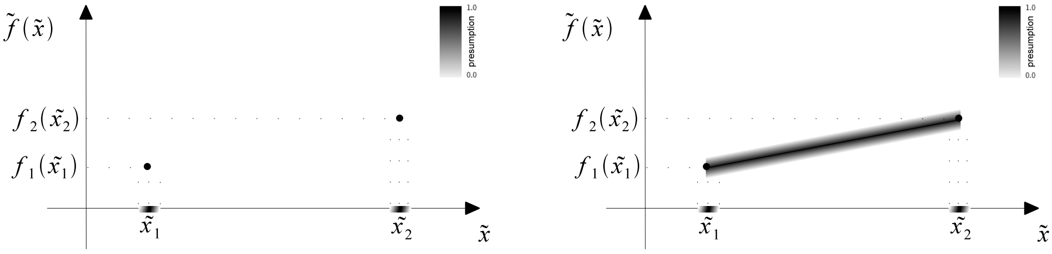

Figure 7: Fuzzy interpolation of data with fuzzy locations and crisp quantities.

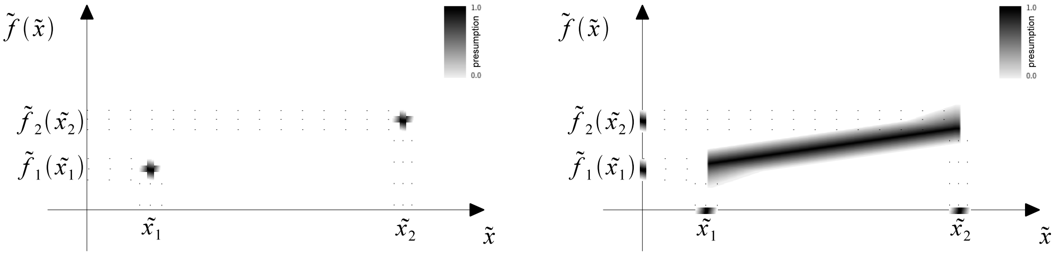

Figure 8: Fuzzy interpolation of data with fuzzy locations and fuzzy quantities.

4. The interpolation problem of data with crisp depth at a crisp location taking into consideration the fuzziness induced by the interpolation itself.

   As mentioned above the uncertainty induced by the interpolation method can be reduced by quantifying the fuzziness and the constructing of a fuzzy number. The Characteristic-Density Factor can be used to modify the resulting fuzzy number to consider the density of the used data set and the characteristics of the studied region.

   Here, considering the locations and the corresponding quantities as crisp data $(x_i, f_i(x_i))$ the basis functions used to build the interpolating function are fuzzy functions $\widetilde{\phi}(\widetilde{x}(x))$, which depend on $\widetilde{x}(x)$. The function $\widetilde{x}(x)$ maps every x to a fuzzy number that quantifies the fuzziness and hence the uncertainty induced by the interpolation method.

$$\widetilde{\widetilde{f}}(x) = \sum_{i=1}^{n} f_i(x_i) \cdot \widetilde{\phi}_i(\widetilde{x}(x)) \tag{16}$$

   Figure 9 shows a one-dimensional fuzzy linear interpolation of crisp data $(x_1, f_1(x_1))$ and $(x_2, f_2(x_2))$. This interpolant takes the fuzziness induced by the interpolation method into account.

   Each of the first three problems gives, if combined with the fourth one, an extra special fuzzy interpolation problem. Regarding the first interpolation problem

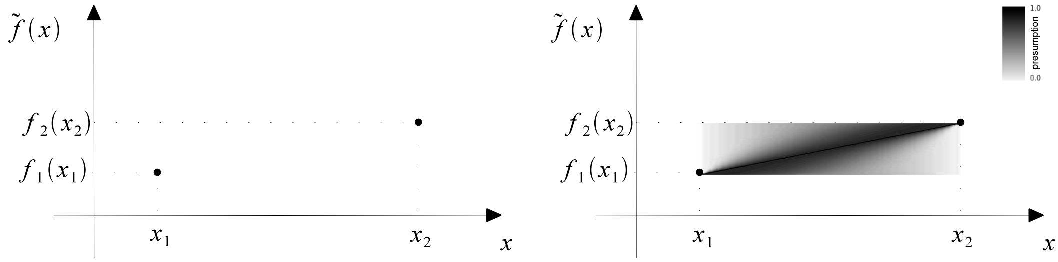

Figure 9: Fuzzy linear interpolation of crisp data.

with fuzzy quantities and crisp locations and introducing the fuzziness induced by the interpolation , the interpolating function is given as

$$\widetilde{\widetilde{f}}(x) = \sum_{i=1}^{n} \widetilde{f}_i(x_i) \cdot \widetilde{\phi}_i(\widetilde{x}(x)) \tag{17}$$

Figure 10 shows a one-dimensional fuzzy interpolation of data with crisp locations and fuzzy interpolated quantities $(x_1, \widetilde{f}_1(x_1))$ and $(x_2, \widetilde{f}_2(x_2))$ considering the fuzziness induced by the interpolant.

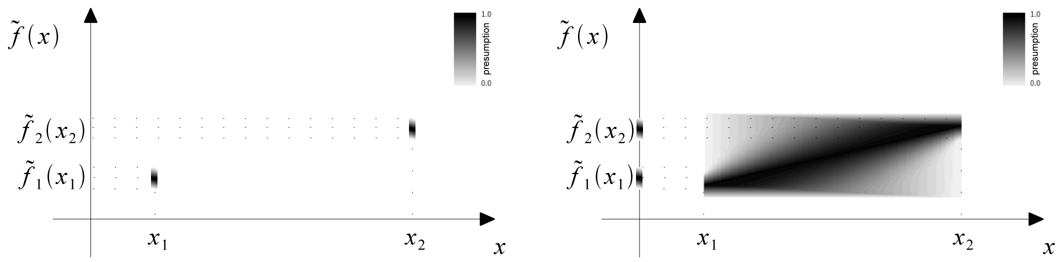

Figure 10: Fuzzy interpolation considering the fuzziness induced by the interpolant.

The combination of the second and third interpolation problem of fuzzy data with the interpolation problem of the fuzziness induced by the interpolation method gives a fuzzy interpolant of type 2. This means that the resulting fuzzy numbers are of type 2. Fuzzy numbers of type 2 express the fuzziness in the presumption itself by considering degrees of presumptions as fuzzy numbers.

The resulting interpolant for the case of fuzzy locations $\widetilde{x}_i$ and crisp interpolated quantities $f_i(\widetilde{x}_i)$ is given as

$$\widetilde{\widetilde{\widetilde{f}}}(x) = \sum_{i=1}^{n} f_i(\widetilde{x}_i) \cdot \widetilde{\widetilde{\phi}}_i(\widetilde{\widetilde{x}}(x, \widetilde{x}_1, \widetilde{x}_2)) \tag{18}$$

The interpolant for the case of fuzzy data with fuzzy locations $\widetilde{x}_i$ and fuzzy quantities $\widetilde{f}_i(\widetilde{x}_i)$ is given as

$$\widetilde{\widetilde{f}}(x) = \sum_{i=1}^{n} \widetilde{f}_i(\widetilde{x}_i) \cdot \widetilde{\widetilde{\phi}}_i(\widetilde{\widetilde{x}}(x, \widetilde{x}_1, \widetilde{x}_2)) \tag{19}$$

5 Fuzzy Digital Bathymetric Model in Space and Time

Representing the Digital Bathymetric Model by a continuous function $z(x, y, t)$ in space and time describes the evolution of the sea floor over time consistently. In [3] the Digital Bathymetric Model was redefined as a collection of discrete survey points and the associated interpolation and interpretation procedures. This definition is adapted here to build the fuzzy Digital Bathymetric Model, although the interpolation methods used are fuzzy.

Two categories of interpolation methods are used here to build the Digital Bathymetric Model. The first one are the mesh-free interpolation methods, for example the inverse distance interpolation that is well known as Shepard Interpolation. The second one are the mesh-based interpolation methods, which divide the studied area into triangles or rectangles. In addition to the spatial interpolations temporal interpolations were introduced in [3] and are adapted here, too. A temporal linear interpolation between two spatially interpolated depths from the directly in time previous and subsequent data sets can be conducted. Polynomial or mesh-free interpolation procedures can be used, should there be extra data sets other than these in time directly adjacent data sets. An optimization of the mesh-free interpolation was suggested in [3] to reduce the resulting effect of smoothing the surface. An appropriate circumsphere in time and space was recommended.

6 Analysis of Morphodynamics

Since the Digital Bathymetric Models were understood as three-dimensional continous interpolating functions of the form $z(x, y, t)$ in space and time many different procedures for the analysis of the morphodynamics were developed in [3] taking advantage of this representation. An inverse finite volume procedure was developed to compute the sedimentation transport out of the bathymetric model. The morphological velocities were introduced to describe the transformation of the local structures, such as the move of tidal channels or the fall of coastal lines.

In this paper the analysis of the morphodynamics is restricted to determining the sedimentation and erosion rates. The other analysis procedures presented in [3] are still under development for the case of the fuzzy digital baythymetric model. The changing rates of the depth (dz/dt) can be determined by using a finite difference scheme on rays parallel to the time axis and thus areas of erosion and sedimentation can be identified.

7 Application

The proposed presentation of a fuzzy Digital Bathymetric Model is applied to bathymetric data sets that cover the offshore area of the island of Langeoog off the German coast of the north sea. Regular bathymetric surveys, in time intervals of one to two years, were conducted by the Lower Saxony Water Management, Coastal Defence and Nature Conservation Agency (NLWKN). The used data acquisition techniques were sonar and laser scanning. The basic data sets, that are used here, consist of a regular grid of 5 m distance and were resulting from laser scan over the years 2002 and 2003. These data sets are supplied by the (NLWKN) after treatment and processing which increases the uncertainty about them.

In this demonstrated application the impreciseness of the depth values, the uncertainty induced by the interpolation method, the density of the data set and the characteristic of the investigated area are used to build a spatial temporal fuzzy bathymetric model. The impreciseness of the location is neglected. The interpolation procedure is a fuzzy bilinear interpolation based on the supplied grid. This model serves as a basis for further analysis. The fuzzy erosion and sedimentation rates are identified as an example of fuzzy analysis of morphodynamics.

Figure 11, 12 and 13 show the fuzzy spatial interpolation of the depth in the Spring of the year 2002. In Figure 11 on the left side the minimum of the depth distribution at the presumption level of (0.0) is presented. The right picture shows the maximum of the depth distribution at the presumption level of (0.0) . In Figure 12 on the left side the minimum of the depth distribution at the presumption level of (0.5) is presented. The right picture shows the maximum of the depth distribution at the presumption level of (0.5) . Figure 13 shows the depth distribution at the presumption level of (1.0).

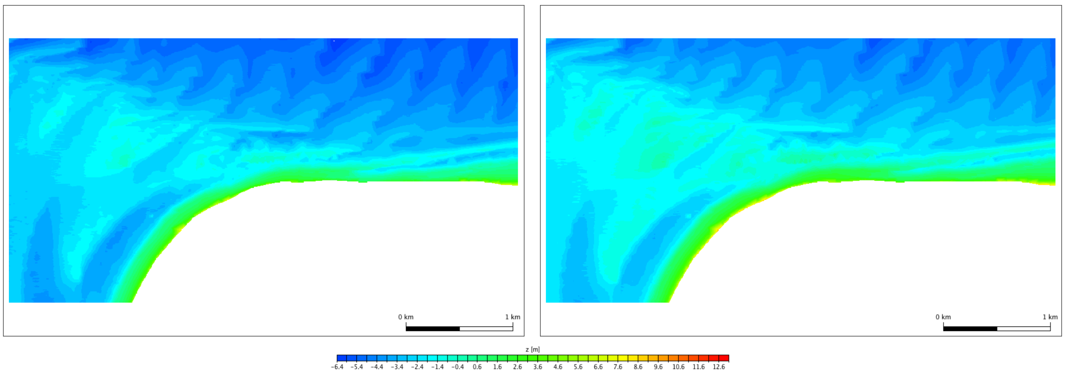

Figure 11: From left to right the minimum and maximum of the spatially interpolated depth distribution in Spring 2002 at 0.0 degree of presumption.

Figure 14, 15 and 16 show the fuzzy spatial interpolation of the depth in the Spring of the year 2003. In Figure 14 on the left side the minimum of the depth distribution at the presumption level of (0.0) is presented. The right picture shows the maximum

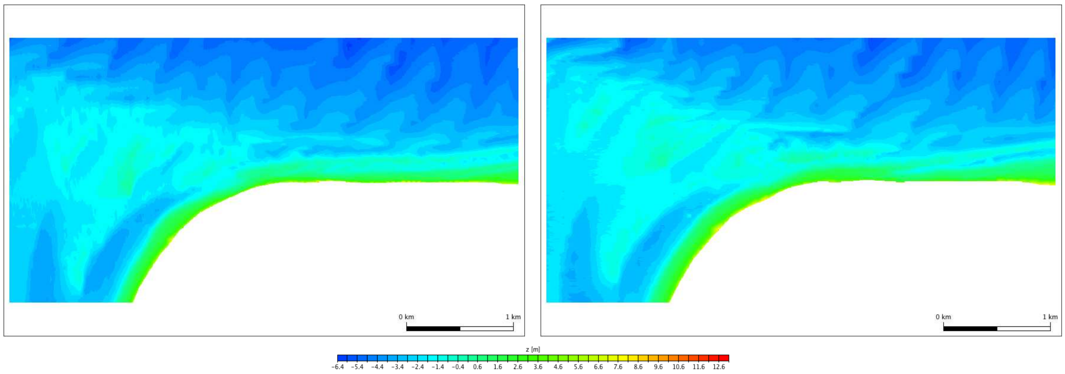

Figure 12: From left to right the minimum and maximum of the spatially interpolated depth distribution in Spring 2002 at 0.5 degree of presumption.

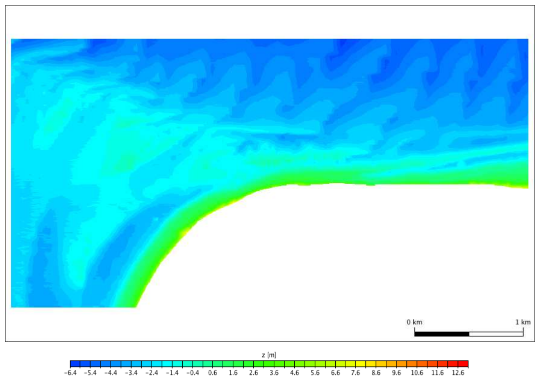

Figure 13: The spatially interpolated depth distribution in Spring 2002 at 1.0 degree of presumption.

of the depth distribution at the presumption level of (0.0) . In Figure 15 on the left side the minimum of the depth distribution at the presumption level of (0.5) is presented. The right picture shows the maximum of the depth distribution at the presumption level of (0.5) . Figure 16 shows the depth distribution at the presumption level of (1.0).

Figure 17, 18 and 19 show the fuzzy spatial-temporal interpolation of the depth in the late Summer of the year 2002. In Figure 17 on the left side the minimum depth distribution at the presumption level of (0.0) is presented. The right picture shows the maximum depth distribution at the presumption level of (0.0) . In Figure 18 on the left side the minimum depth distribution at the presumption level of (0.5) is presented. The right picture shows the maximum depth distribution at the presumption level of (0.5) . Figure 19 shows the depth distribution at the presumption level of (1.0).

Figure 20, 21 and 22 show the fuzzy spatial-temporal interpolation of the depth in the early Winter of the year 2002. In Figure 20 on the left side the minimum depth distribution at the presumption level of (0.0) is presented. The right picture shows the

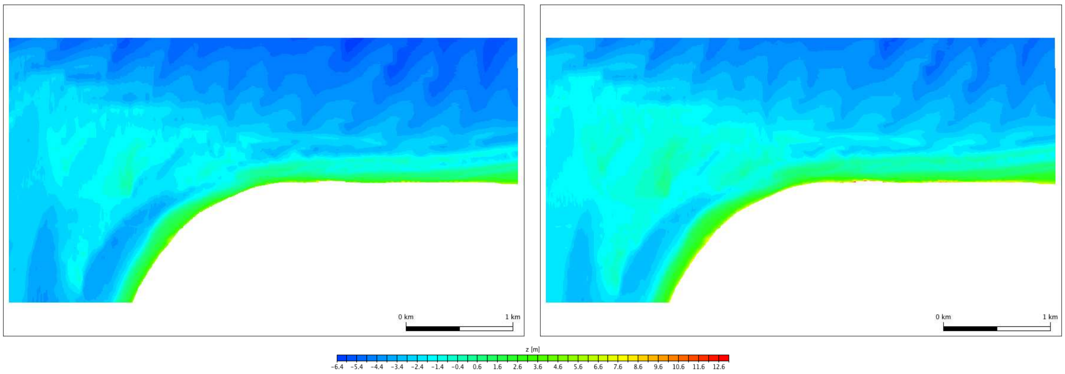

Figure 14: From left to right the minimum and maximum of the spatially interpolated depth distribution in Spring 2003 at 0.0 degree of presumption.

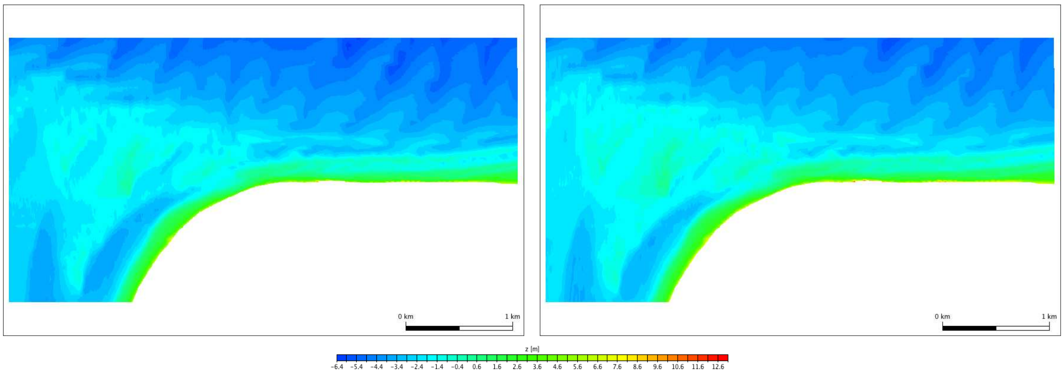

Figure 15: From left to right the minimum and maximum of the spatially interpolated depth distribution in Spring 2003 at 0.5 degree of presumption.

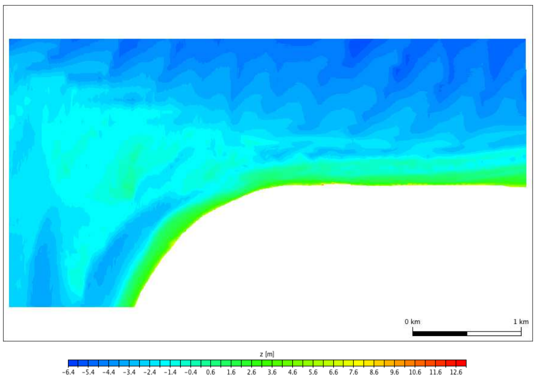

Figure 16: The spatially interpolated depth distribution in Spring 2003 at 1.0 degree of presumption.

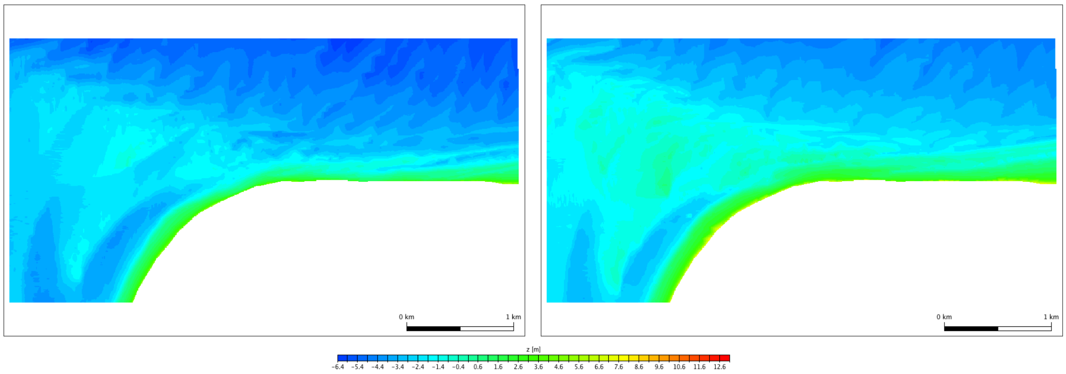

Figure 17: From left to right the minimum and maximum of the spatially and temporally interpolated depth distribution in late Summer 2002 at 0.0 degree of presumption.

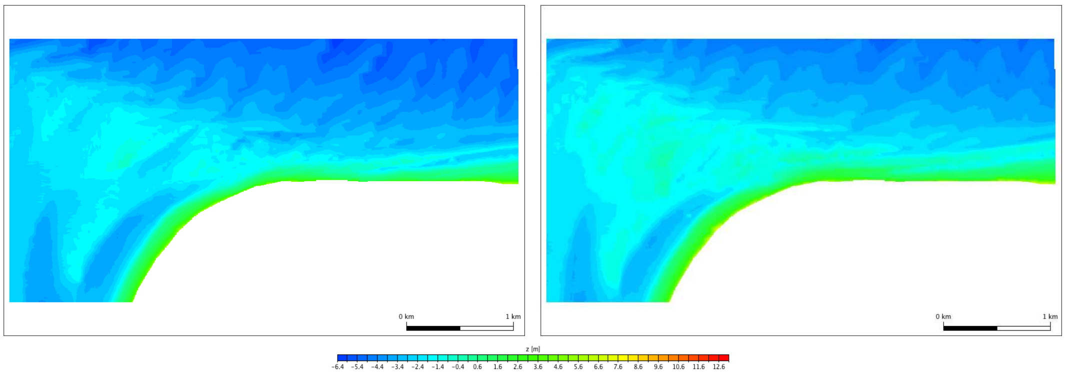

Figure 18: From left to right the minimum and maximum of the spatially and temporally interpolated depth distribution in late Summer 2002 at 0.5 degree of presumption.

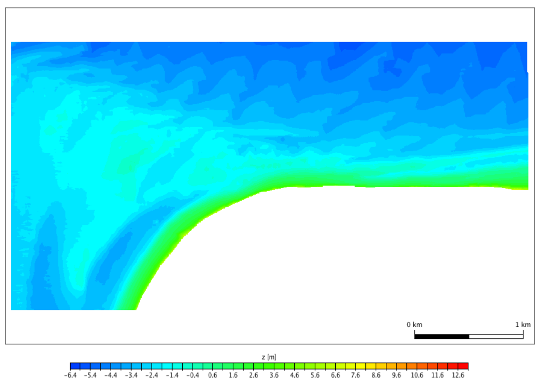

Figure 19: The spatially and temporally interpolated depth distribution in late Summer 2002 at 1.0 degree of presumption.

maximum depth distribution at the presumption level of (0.0) . In Figure 21 on the left side the minimum depth distribution at the presumption level of (0.5) is presented. The right picture shows the maximum depth distribution at the presumption level of (0.5) . Figure 22 shows the depth distribution at the presumption level of (1.0).

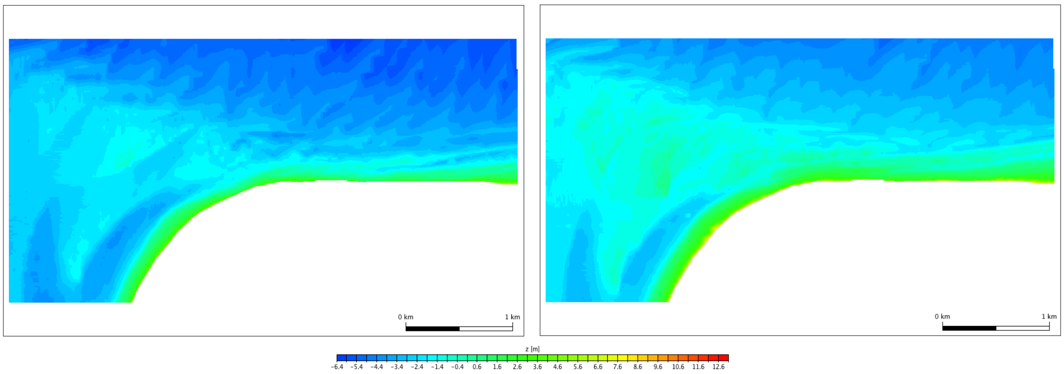

Figure 20: From left to right the minimum and maximum of the spatially and temporally interpolated depth distribution in early Winter 2002 at 0.0 degree of presumption.

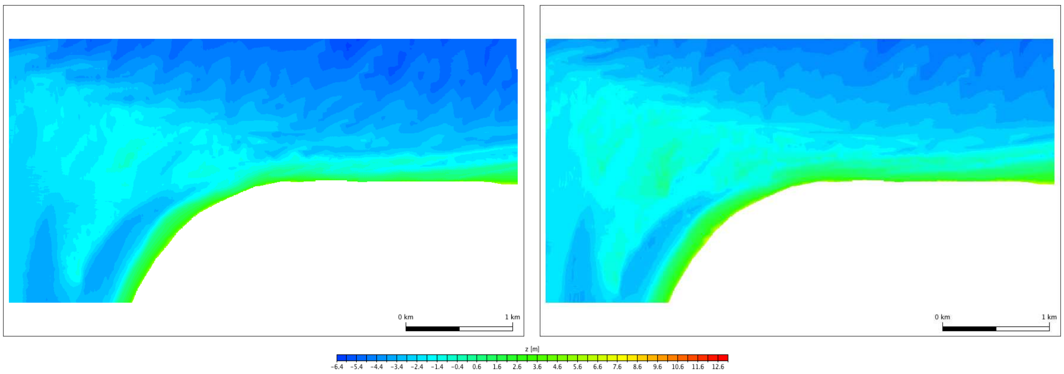

Figure 21: From left to right the minimum and maximum of the spatially and temporally interpolated depth distribution in early Winter 2002 at 0.5 degree of presumption.

Figure 23, 24 and 25 show the resulting fuzzy erosion and sedimentation rates between the years 2002 and 2003. In Figure 23 on the left side the minimum depth change at the presumption level of (0.0) is presented. The right picture shows the maximum depth change at the presumption level of (0.0) . In Figure 24 on the left side the minimum depth change at the presumption level of (0.5) is presented. The right picture shows the maximum depth change at the presumption level of (0.5) . Figure 25 shows the depth change at the presumption level of (1.0).

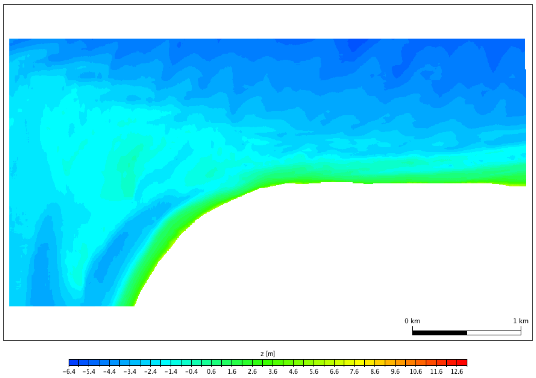

Figure 22: The spatially and temporally interpolated depth distribution in early Winter 2002 at 1.0 degree of presumption.

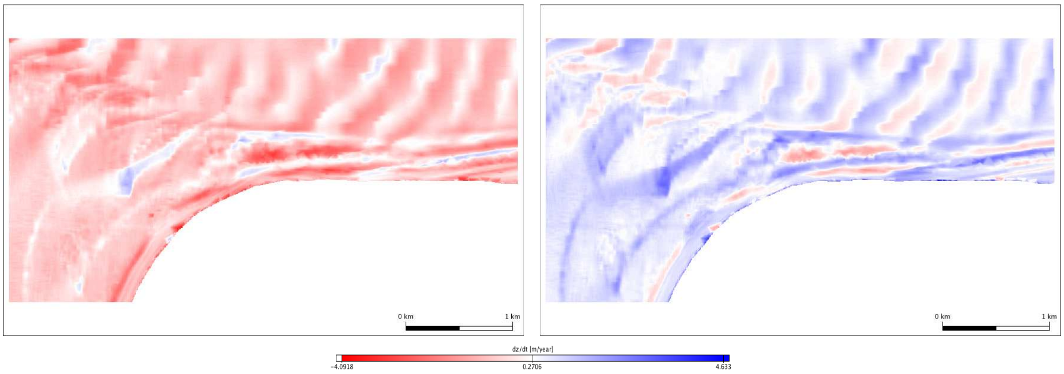

Figure 23: From left to right the minimum and maximum depth change at 0.0 degree of presumption.

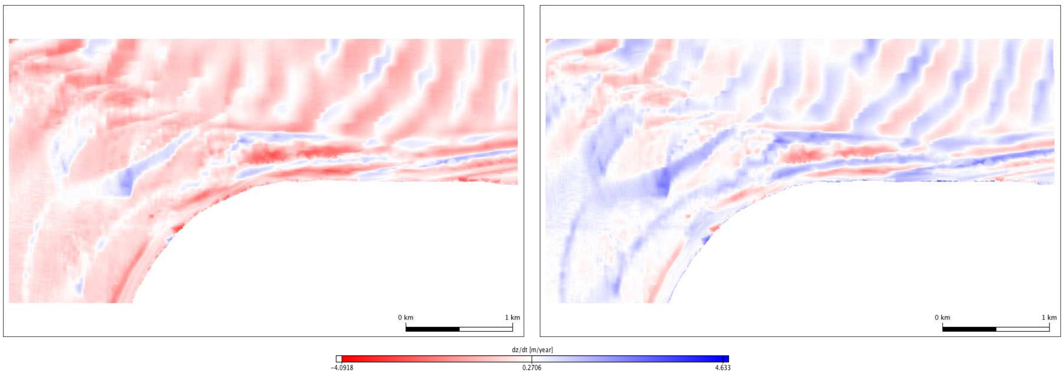

Figure 24: From left to right the minimum and maximum depth change at 0.5 degree of presumption.

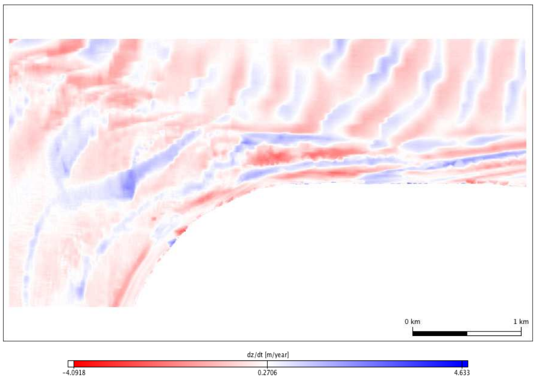

Figure 25: The depth change at 1.0 degree of presumption.

8 Conclusion and Outlook

In this paper a fuzzy Digital Bathymetric Model was introduced. The sources of uncertainty in the process of building a spatial-temporal Digital Bathymetric Model were presented. The construction of fuzzy numbers from the sources of uncertainty was discussed. The different cases of fuzzy interpolation were a subject of discussion. The possible techniques of analysis of morphodynamics and the one adopted here were highlighted. The proposed Digital Bathymetric Model has been implemented and the practicability was shown on bathymetric data sets that cover the offshore area of the island of Langeoog off the German coast of the north sea.

Spatial-temporal fuzzy interpolation and analysis are shown to extend spatial-temporal interpolation to model the morphodynamic processes in a more comprehensive way. They guarantee a complete interpretation of all available information on the measured basic data. For instance, one can derive at any arbitrary point of time a consistent fuzzy Digital Bathymetric Model, with which the deduced uncertainty is quantified at every location. The rates of erosion and sedimentation with their associated uncertainty can then be quantified in a conceivable way.

However, the practical challenge of the fuzzy spatial-temporal interpolations must be well elaborated and improved. The techniques of analysis of morphodynamics introduced in [3] must be expanded to cover the fuzzy case.

References

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