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Directional Wave Propagation and Induced Currents in Coastal Regions

Peter Milbradt

Abstract:

This paper describes a spectral wind-wave model, which explains the propagation, growth and decay of short-period waves in nearshore areas. The model includes the effects of refraction and shoaling due to varying depth und currents, wave generation caused by wind and energy dissipation due to bottom friction and wave breaking.

The instationary model is solved by the Finite Element Method and is implemented in Java. Several examples prove the correctness of the implementation and show possible fields of applications.

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Introduction

The sea surface in the presence of wind waves is irregular and the propagation of waves in the coastal zone is transformed due to currents and irregular water depths. The wave-current interaction is one of the most interesting and important phenomena for the prediction of wave climate and resultant sediment transport in coastal areas. The approximation of irregular waves by monochromatic waves in modeling of wave transformation in coastal areas is very efficient but often introduces large errors in wave heights.

The presented wave-current model has been developed for the simulation and prediction of waves and currents in a large area including the transformation depending on shallow water conditions. A holistic view is introduced for the numerical modeling of the interaction between irregular waves and currents.

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Wave Model

The wave model is based on an Eulerian formulation of the discret spectral balance of action density $N(\sigma,\theta)$ which is equal to the energy density $E(\sigma,\theta)$ divided by the relative frequency σ : $N(\sigma,\theta) := E(\sigma,\theta)/\sigma$. The action density is a function of the geographical (x,y) space and the spectral space (σ -relative frequency, θ -propagation direction). The kinematic behaviour of the waves is described by the linear theory for surface gravity waves (including the effects of currents).

The rate of change of action density $N(\sigma,\theta)$ for irregular waves is described by the spectral action conservation equation in the general form

$$\frac{\partial N}{\partial t} + \nabla(\vec{c}N) + \frac{\partial c_\theta N}{\partial \theta} + \frac{\partial c_\sigma N}{\partial \sigma} - \frac{S}{\sigma} - \frac{D}{\sigma} - \frac{R}{\sigma} = 0.$$

Here $\vec{c}$, c_θ and c_σ are the diffenent rates of changes in geographical and spectral space. The term S is the energy supplied to the spectrum by the effect of local winds, the term D represents energy subtraction from the spectrum by bottom friction and wave breaking and the term R is the energy transfer between waves and currents.

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Characteristic Velocities

Using the linear wave theory the following rates of changes can be obtained in geographical and spectral space.

The propagation velocity $\vec{c}$ of wave energy in geographical x -, y -space is:

$$\vec{c} := \vec{C_g} + \vec{U},$$

where $\vec{U}$ is the vector of current velocity and $\vec{C_g}$ is the group velocity vector.

The new formulation of the propagation velocity c_θ in θ -direction is based on the hyperbolic wave model developed in [5]

$$c_\theta := \frac{C_g}{k}\left(\frac{\partial k_x}{\partial y} - \frac{\partial k_y}{\partial x}\right) - \frac{\vec{k}}{k} \cdot \frac{\partial \vec{U}}{\partial m},$$

where m is a spatial coordinate perpendicular to the propagation direction, k the wave number and $\vec{k} = (k_x, k_y)$ its vector.

Frequency shifting c_σ due to currents and nonstationary depth distribution can be implemented in the following form:

$$c_\sigma := \frac{\partial \sigma}{\partial d}\left[\frac{\partial d}{\partial t} + \vec{U} \cdot \nabla d\right] - C_g \vec{k} \cdot \frac{\partial \vec{U}}{\partial s},$$

where s is the space coordinate in the propagation direction θ and d the total depth.

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Energy Input by Wind

The mechanism of energy transfer from the air to the waterbody are described by Philips[7] and Miles[6]

$$S = \alpha + \beta \cdot E(f, \theta)$$

The α term initiates wave growth but is unimportant thereafter. A simple form was given by Philips[8]

$$\alpha\,(k) = \frac{80 \cdot \rho_a^2 \cdot \sigma}{\rho_w^2 \cdot g^2 \cdot k^2} \cdot C_D^2 \cdot u^4.$$

Here $\rho_{a,w}$ are air and water density, C_D is the drag coefficient ($C_D = 0.0012$) and u is the wind componente along the ray direction.

The term β gives exponetial growth. The formulation from Hsiao und Shemdin [3] are used:

$$\beta = 0.12 \frac{\rho_a}{\rho_w} \sigma \left(\frac{8}{3\pi} \frac{u}{C} - 1 \right)^2$$

where C is the wave velocity.

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Wave Breaking

To describe the energy dissipation and the input of turbulent energy due to wave breaking in the water body, a lot of breaking criteria can be realized. The energy loss is described by the energy dispersion coefficient ϵ_b included in the dissipation term D. Following the Dally, Dean and Dalrymple[1] model there is a stable wave height after breaking:

$$\epsilon_b(E_{ij}) = -\frac{K_B}{d}\left(E_{ij}C_g - (E_{ij}C_g)_s\right)$$

where E_{ij} is a energy spectral componet and K_B is a parameter.

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Radiation Stresses

The interaction of currents and waves in shallow water regions are neglected in the most spectral wave models in the literature. As a first approximation of the full interaction between waves and currents the concept of radiation stresses [4] are using:

$$R = \sum_{i=1}^{2} \left(\sum_{j=1}^{2} S_{ij} \frac{\partial U_j}{\partial x_i} U_i (T_i - T_i^B) \right) .$$

For each spectral component a radiation stress tensor can be computed.

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Current Model

The current model is based on the shallow water equations. The interaction between currents and waves are modeled by the concept of radiation stresses.

$$\frac{\partial U_x}{\partial t} = -U_x \frac{\partial U_x}{\partial x} \; U_y \frac{\partial U_x}{\partial y} - g \frac{\partial \eta}{\partial x} - \frac{1}{\rho_w(\eta + h)} \frac{\partial S_{xj}(E)}{\partial x_j} + \frac{1}{\rho_w(\eta + h)} (T_x - T_x^B) \qquad (1)$$

$$\frac{\partial U_y}{\partial t} = -U_x \frac{\partial U_y}{\partial x} \; U_y \frac{\partial U_y}{\partial y} - g \frac{\partial \eta}{\partial y} - \frac{1}{\rho_w(\eta + h)} \frac{\partial S_{yj}(E)}{\partial x_j} + \frac{1}{\rho_w(\eta + h)} (T_y - T_y^B) \qquad (2)$$

$$\frac{\partial \eta}{\partial t} = -\frac{\partial U_x(\eta + h)}{\partial x} - \frac{\partial U_y(\eta + h)}{\partial y} \qquad (3)$$

In these equations $\vec{U}$ stands for the vertically integrated velocities, h for the water depth at rest, and η for the free surface. The earth acceleration is g, ρ_w is the water density, represents turbulent effects and T^B the bottom friction. Commonly the Taylor-Newton formulation is used, to which the influence of orbital motion of the waves is added.

The resultant radiation stress term S_{ij} represents the momentum flux induced by the deformation of short period waves running at the surface of the water body. These effects are very small for deep water conditions, however, they are important in regions of deformation and breaking of short waves. These terms couple long and short wave propagation.

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The Numerical Model

The model is formulated in Cartesian coordinates (i.e., in a flat plane) which are acceptable considering the spatial scale of coastal areas. The wave field at each point is described by a discrete spectrum. The spectral resolution is variable and will be selected depending on the problem.

The integration of the instationary basic equations, action conservation and shallow water equations

$$\frac{\partial N_{ij}}{\partial t} = -\nabla(\vec{c} \cdot N_{ij}) - \frac{\partial}{\partial \theta}\{c_\theta N_{ij}\} - \frac{\partial}{\partial \sigma}\{c_\sigma N_{ij}\} + \frac{S}{\sigma} + \frac{D}{\sigma} + \frac{R}{\sigma}$$

and

$$\frac{\partial U_x}{\partial t} = -U_x \frac{\partial U_x}{\partial x} U_y \frac{\partial U_x}{\partial y} - g \frac{\partial \eta}{\partial x} - \frac{1}{\rho(\eta + h)} \frac{\partial S_{xj}(E)}{\partial x_j} + \frac{1}{\rho(\eta + h)}(T_x - T_x^B)$$

$$\frac{\partial U_y}{\partial t} = -U_x \frac{\partial U_y}{\partial x} U_y \frac{\partial U_y}{\partial y} - g \frac{\partial \eta}{\partial y} - \frac{1}{\rho(\eta + h)} \frac{\partial S_{yj}(E)}{\partial x_j} + \frac{1}{\rho(\eta + h)}(T_y - T_y^B)$$

$$\frac{\partial \eta}{\partial t} = -\frac{\partial U_x(\eta + h)}{\partial x} - \frac{\partial U_y(\eta + h)}{\partial y}$$

is based on the finite element method in the geographical space and on finite differences in the spectral space. Triangular grids are used with linear interpolation functions in georgaphical space. The integration is following the stream-line upwinding technique [5]. The upwinding parameters are obtained proportional to the characteristic velocities of the different equations by introducing corresponding Peclet-numbers.

The time integration is preformed in the common step by step manner. Due to the simple structure of the software different 1st to 4th order time integration schemes are implemented.

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Implementation Aspects

The model is implemented in the platform independent programming language Java. With the expected performance of desktop computers as well as the development of the Java runtime environment a reduction of the computing times are to be expected.

Using the Java-class Thread and the inner class construction a simple parallel implementation is realised.

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Numerical Tests and Applications

To validate the implementation of wave propagation, computational results have been compared with solutions from the linear wave theory. Examples of simulations are given for wave propagation and wave-current interaction on a sloping beach.

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Sloping Bottom and a Shoal

In the first example the behaviour of the model over bathymetry, consisting of a sloping bottom with slope 1/50 and a shoal, is studied [2]. No interaction with the current is considered. In the figure 1 the results of a inter-period wave model are shown. Such models are able to describe the combined effects of refraction, diffraction and reflection on wave transformation. In the test area one can see crossing waves behind the shoal. The discretisation of inter-period wave model is very high (10 grid points per wave length). Period-avarage models like a hyperbolic wave model [5] which is useable for large areas can not describe crossing waves. The presented spectral wave model describes irregular waves, crossing waves (see figure 2) and can also used in larger areas.

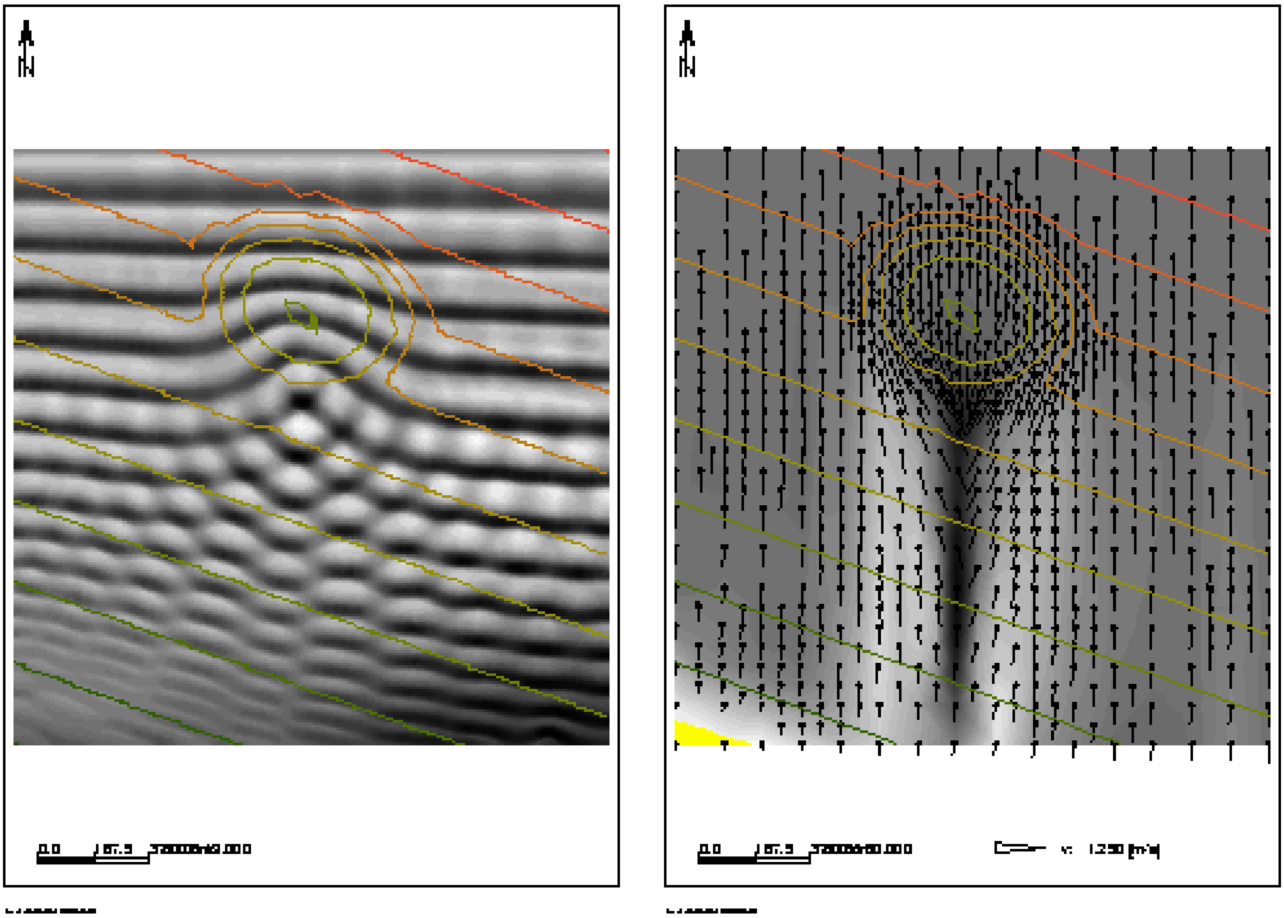

Figure 1: Results from an inter-period and a hyperbolic wave model

In figure 2 one can see the domain of investigation with the wave spectral at each computational point. The smaller window show zooms of such spectra, the surface at the top and two transections, one over the frequency (the red or thick line) and one over the direction (the green or thin line). As a result of this simulation one can see that very small and short frequency spectral components can undisturbedly propagate over the shoal. Behind the shoal the wave spectra have two extrema in their direction distribution, one from the left and one from the right side of the shoal. Not only the quality of the reproduction of the physical phenomena but also the computation time of a spectral model lies between that of a wave-resolving and the hyperbolic model.

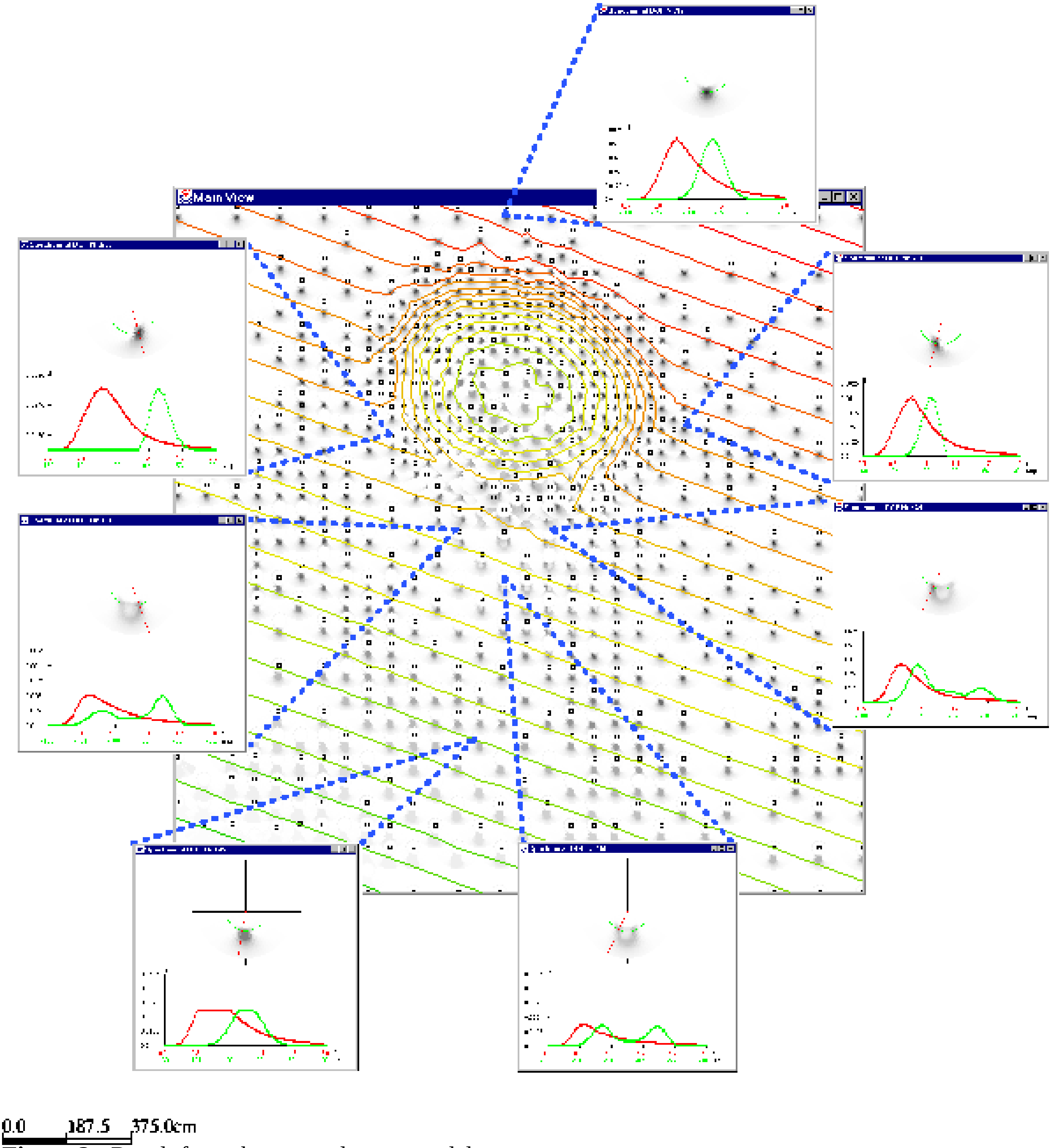

Figure 2: Result from the spectral wave model

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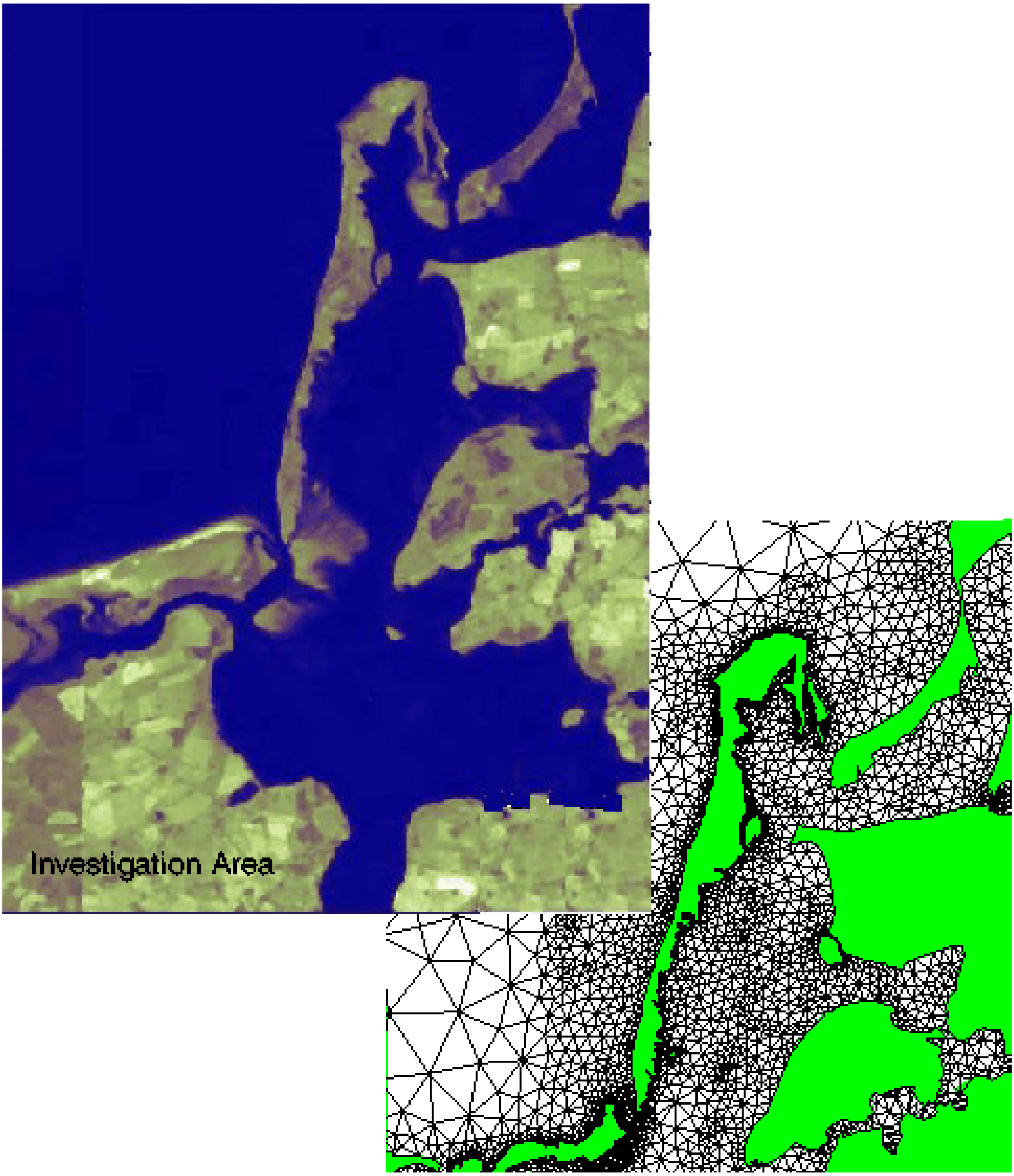
German Baltic Sea coast

A more complex investigation area is given at the german coast of the Baltic Sea near the island of Rügen. The non-tidal coastal areas of the Baltic Sea are formed mainly by wave forces and wave induced processes. The sandbank `¨Bock`¨ is of major concern in a project named MorWin (see [10]) and is located at the south tip of the island Hiddensee and is seperated by the Gellen strait. Two smaller channels bifurcate from the Gellen-Barhöfter strait to the Bodden. In figure 3 one can see the domain of investigation and a part of the finite element grid. It should be underlined that the use of the finite element method will give local improvement especially in those ranges which are of interest.



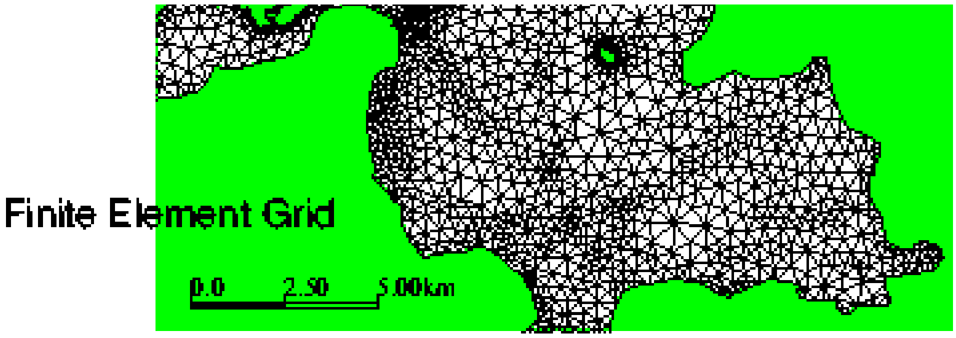
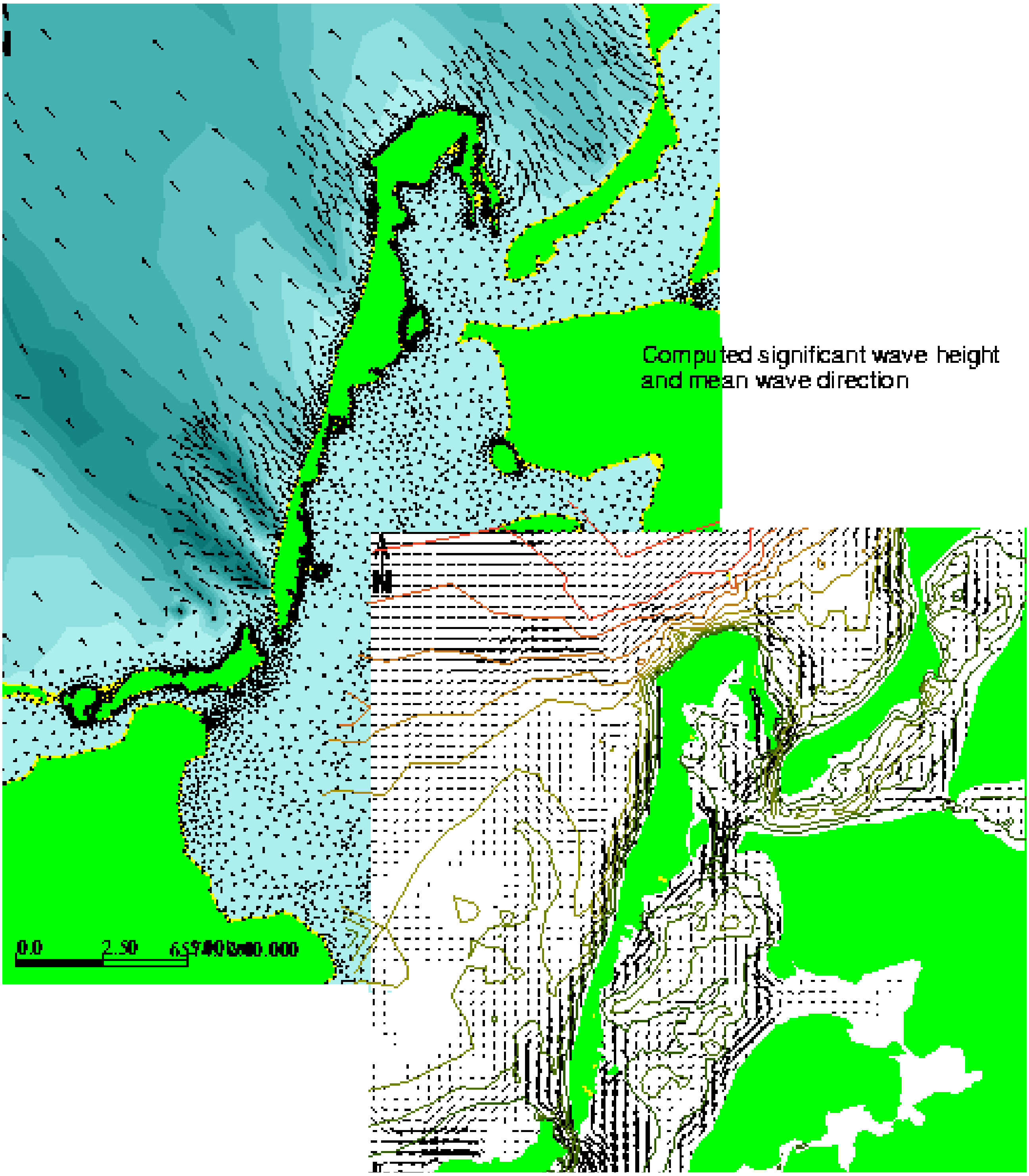


Figure 3: Bathymetry and Finite Element Grid

Figure 4 shows a results from simulation of a west wind situation. The plotted significant wave height and mean wave direction shows a focussing of wave energy at the south tip of the island of Hiddensee and near the sandbank ``Bock". This simulation has also computed current velocities driven by waves and by the boundary conditions (specified by a hindcast data).



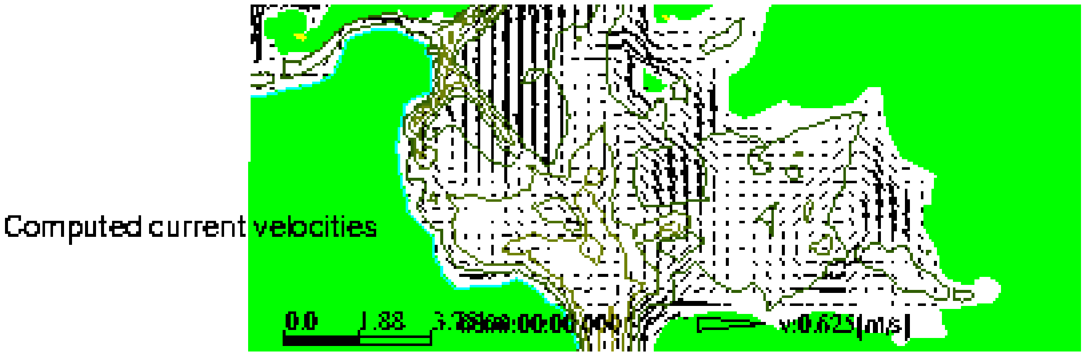


Figure 4: Results

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Conclusion

It is impossible to give an exact mapping of the natural processes in full complexity. However in this contribution effective mathematical methods are presented and a platform independent simulation tool is given for a systematical and reproducible approximations of the wave-current interaction for irregular waves.

The presented model allows modeling of irregular waves and currents in large areas thus describing near shore processes. A closed set of differential equations is used for irregular waves and vertically integrated currents. The set of equations is of propagation type. This allows for applications of the same numerical scheme, wich is based on linear finite elements and an upwindig Petrov-Galerkin procedure, to all equations simultanously.

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